CHAPTER 5

Risk and Return: Past and Prologue

5.1 RATES OF RETURN

Holding Period Return

$$HPR = \frac{P_1 - P_0 + D_1}{P_0}$$

P_0 = Beginning Price

P_1 = Ending Price

D_1 = Cash Dividend

Rates of Return: Single Period Example

Ending Price = 24

Beginning Price = 20

Dividend = 1

HPR = (24 - 20 + 1)/ (20) = 25%

Measuring Investment Returns Over Multiple Periods

- May need to measure how a fund performed over a preceding five-year period
- Return measurement is more ambiguous in this case

Rates of Return: Multiple Period Example Text (Page 128)

Data from Table 5.1

	1	2	3	4
Assets(Beg.)	1.0	1.2	2.0	.8
HPR	.10	.25	(.20)	.25
TA (Before				
Net Flows	1.1	1.5	1.6	1.0
Net Flows	0.1	0.5	(0.8)	0.0
End Assets	1.2	2.0	.8	1.0

Returns Using Arithmetic and Geometric Averaging

Arithmetic

$r_a = (r_1 + r_2 + r_3 + ... r_n) / n$

$r_a = (.10 + .25 - .20 + .25) / 4$

$= .10$ or 10%

Geometric

$r_g = \{[(1+r_1) (1+r_2) .... (1+r_n)]\}^{1/n} - 1$

$r_g = \{[(1.1) (1.25) (.8) (1.25)]\}^{1/4} - 1$

$= (1.5150)^{1/4} -1 = .0829 = 8.29\%$

Dollar Weighted Returns

Internal Rate of Return (IRR) - the discount rate that results in present value of the future cash flows being equal to the investment amount

- Considers changes in investment
- Initial Investment is an outflow
- Ending value is considered as an inflow
- Additional investment is a negative flow
- Reduced investment is a positive flow

Dollar Weighted Average Using Text Example (Page 128)

Net CFs	1	2	3	4
$ (mil)	- 0.1	-0 .5	0.8	1.0

$$1.0 = \frac{-0.1}{1+IRR} + \frac{-0.5}{(1+IRR)^2} + \frac{0.8}{(1+IRR)^3} + \frac{1.0}{(1+IRR)^4} = 4.17\%$$

Quoting Conventions

APR = annual percentage rate

(periods in year) X (rate for period)

EAR = effective annual rate

$(1 + \text{rate for period})^{\text{Periods per yr}} - 1$

Example: monthly return of 1%

APR = 1% X 12 = 12%

EAR = $(1.01)^{12} - 1$ = 12.68%

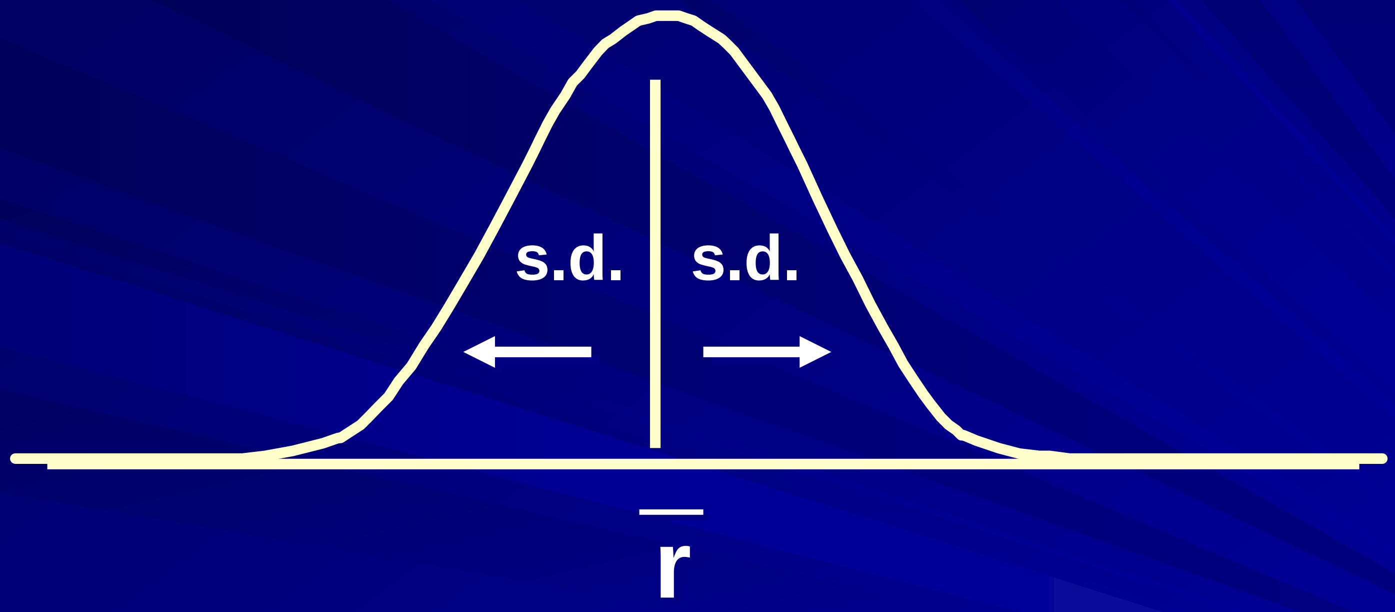
5.2 RISK AND RISK PREMIUMS

Scenario Analysis and Probability Distributions

1) Mean: most likely value
2) Variance or standard deviation
3) Skewness

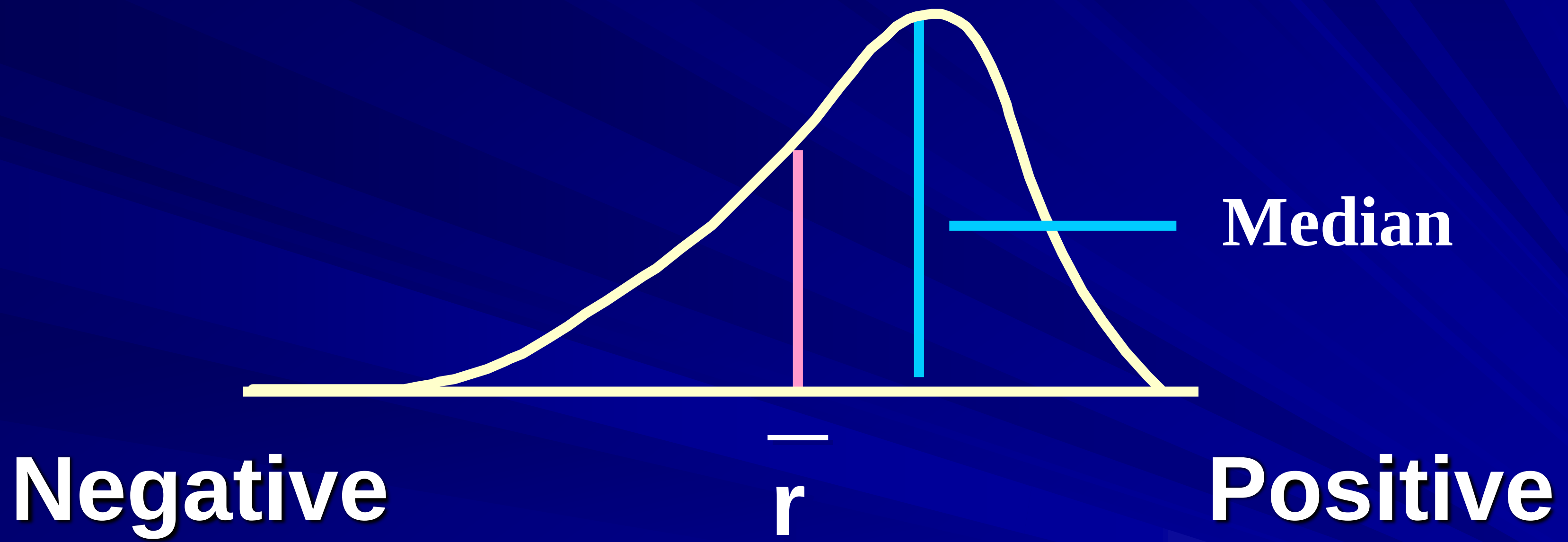
* If a distribution is approximately normal, the distribution is described by characteristics 1 and 2

Normal Distribution

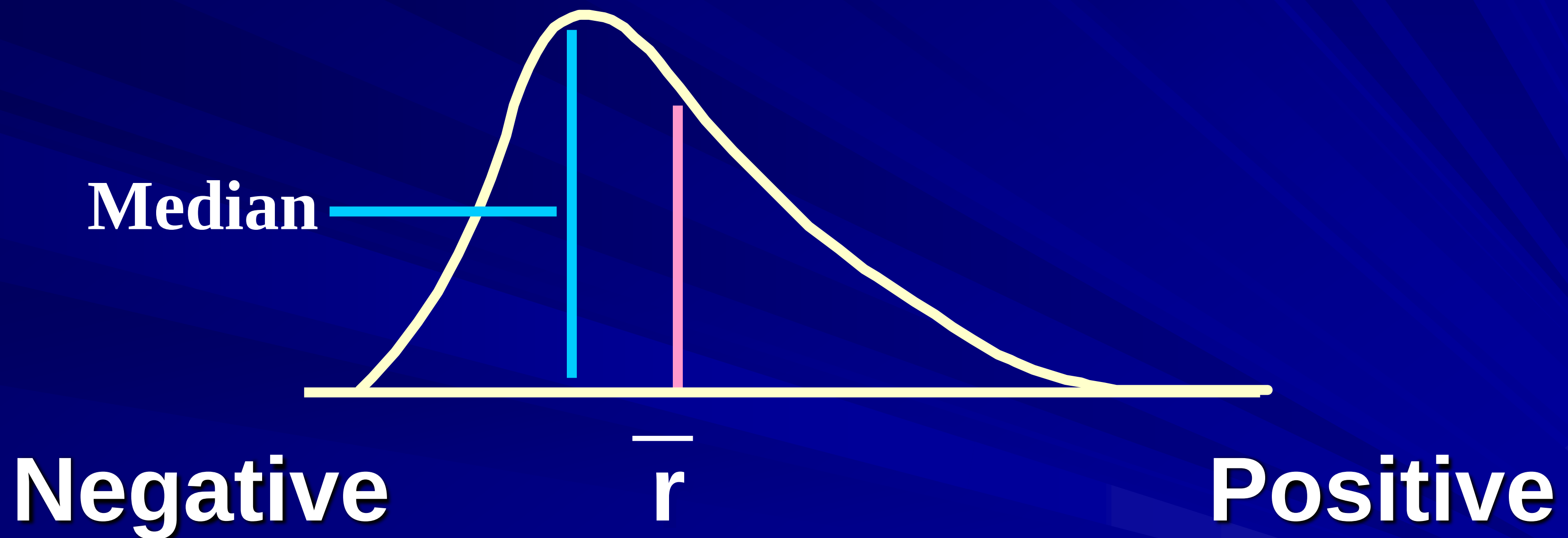


Symmetric distribution

Skewed Distribution: Large Negative Returns Possible



Skewed Distribution: Large Positive Returns Possible



Measuring Mean: Scenario or Subjective Returns

Subjective returns

$$E(r) = \sum_{t=1}^{S} p(s)r(s)$$

p(s) **= probability of a state**
r(s) **= return if a state occurs**
1 to s states

Numerical Example: Subjective or Scenario Distributions

State	Prob. of State	r_{in} State
1	.1	-.05
2	.2	.05
3	.4	.15
4	.2	.25
5	.1	.35

$E(r) = (.1)(-.05) + (.2)(.05)...+ (.1)(.35)$

$E(r) = .15$ or 15%

Measuring Variance or Dispersion of Returns

Subjective or Scenario

$$Var(r) = \sum_{t=1}^{s} p(s)\left[r(s) - E(r)\right]^2$$

$$SD(r) \equiv \sigma = \sqrt{Var(r)}$$

Measuring Variance or Dispersion of Returns

Using Our Example:

Var =[(.1)(-.05-.15)2+(.2)(.05- .15)2...+ .1(.35-.15)2]
Var= .01199
S.D.= [.01199] 1/2 = .1095 or 10.95%

Risk Premiums and Risk Aversion

- Degree to which investors are willing to commit funds
  - Risk aversion
- If T-Bill denotes the risk-free rate, r_f, and variance, σ_P^2, denotes volatility of returns then:

  The risk premium of a portfolio is:

$$E(r_P) - r_f$$

Risk Premiums and Risk Aversion

- To quantify the degree of risk aversion with parameter A:

$$E(r_P) - r_f = \frac{1}{2} A \sigma_P^2$$

  - Or:

$$A = \frac{E(r_P) - r_f}{\frac{1}{2}\sigma_P^2}$$

The Sharpe (Reward-to-Volatility) Measure

$$S = \frac{\text{portfolio risk premium}}{\text{standard deviation of portfolio excess return}}$$

$$= \frac{E(r_P) - r_f}{\sigma_P}$$

5.3 THE HISTORICAL RECORD

Annual Holding Period Returns From Table 5.3 of Text

Series	Geom. Mean%	Arith. Mean%	Stan. Dev.%
World Stk	9.80	11.32	18.05
US Lg Stk	10.23	12.19	20.14
US Sm Stk	12.43	18.14	36.93
Wor Bonds	5.80	6.17	9.05
LT Treas.	5.35	5.64	8.06
T-Bills	3.72	3.77	3.11
Inflation	3.04	3.13	4.27

Annual Holding Period Excess Returns From Table 5.3 of Text

Series	Risk Prem.	Stan. Dev.%	Sharpe Measure
World Stk	7.56	18.37	0.41
US Lg Stk	8.42	20.42	0.41
US Sm Stk	14.37	37.53	0.38
Wor Bonds	2.40	8.92	0.27
LT Treas	1.88	7.87	0.24

Figure 5.1 Frequency Distributions of Holding Period Returns

FIGURE 5.1

Frequency distribution of annual HPRs, 1926–2006

Source: Prepared from data in Table 5.3.

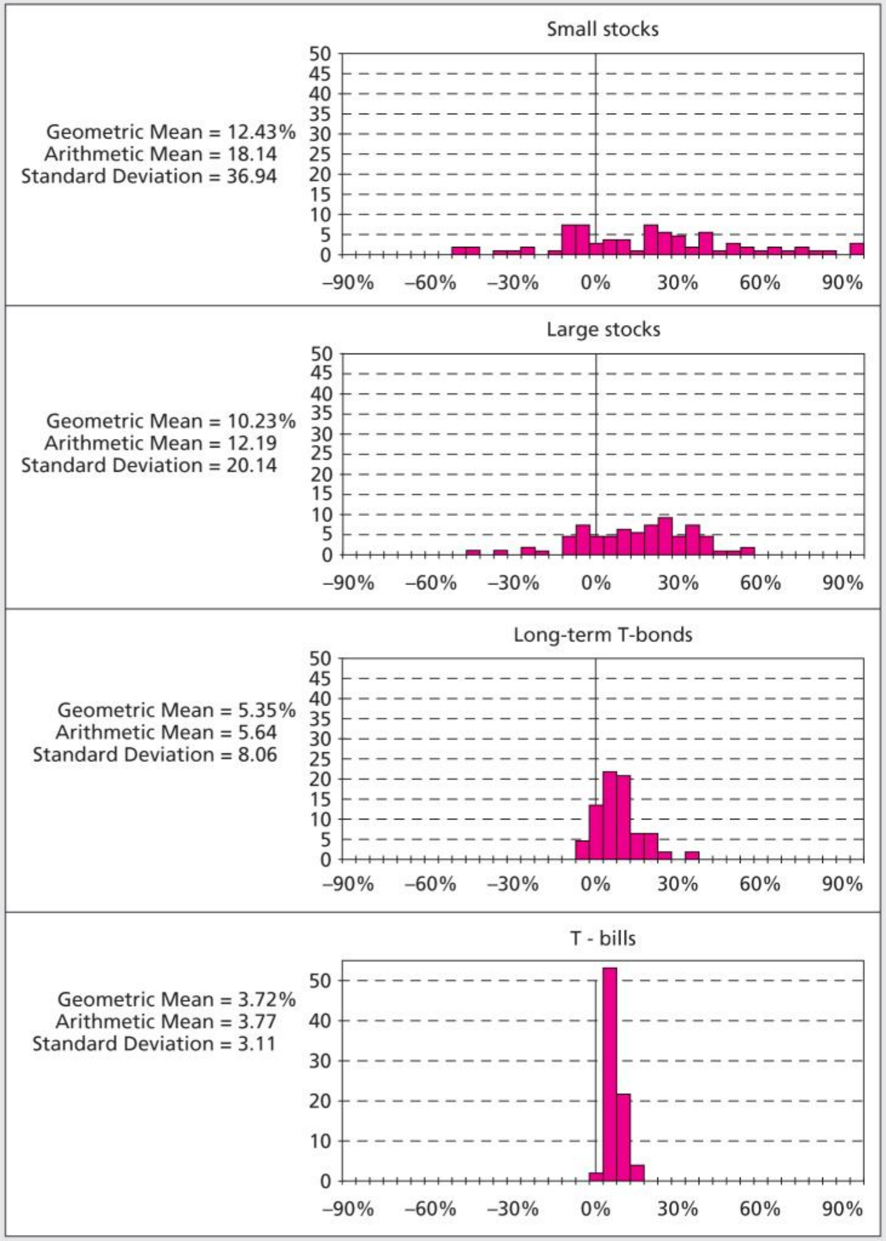


Figure 5.2 Rates of Return on Stocks, Bonds and T-Bills

FIGURE 5.2

Rates of return on stocks, bonds and T-bills, 1926–2006

Source: Prepared from Table 5.3.

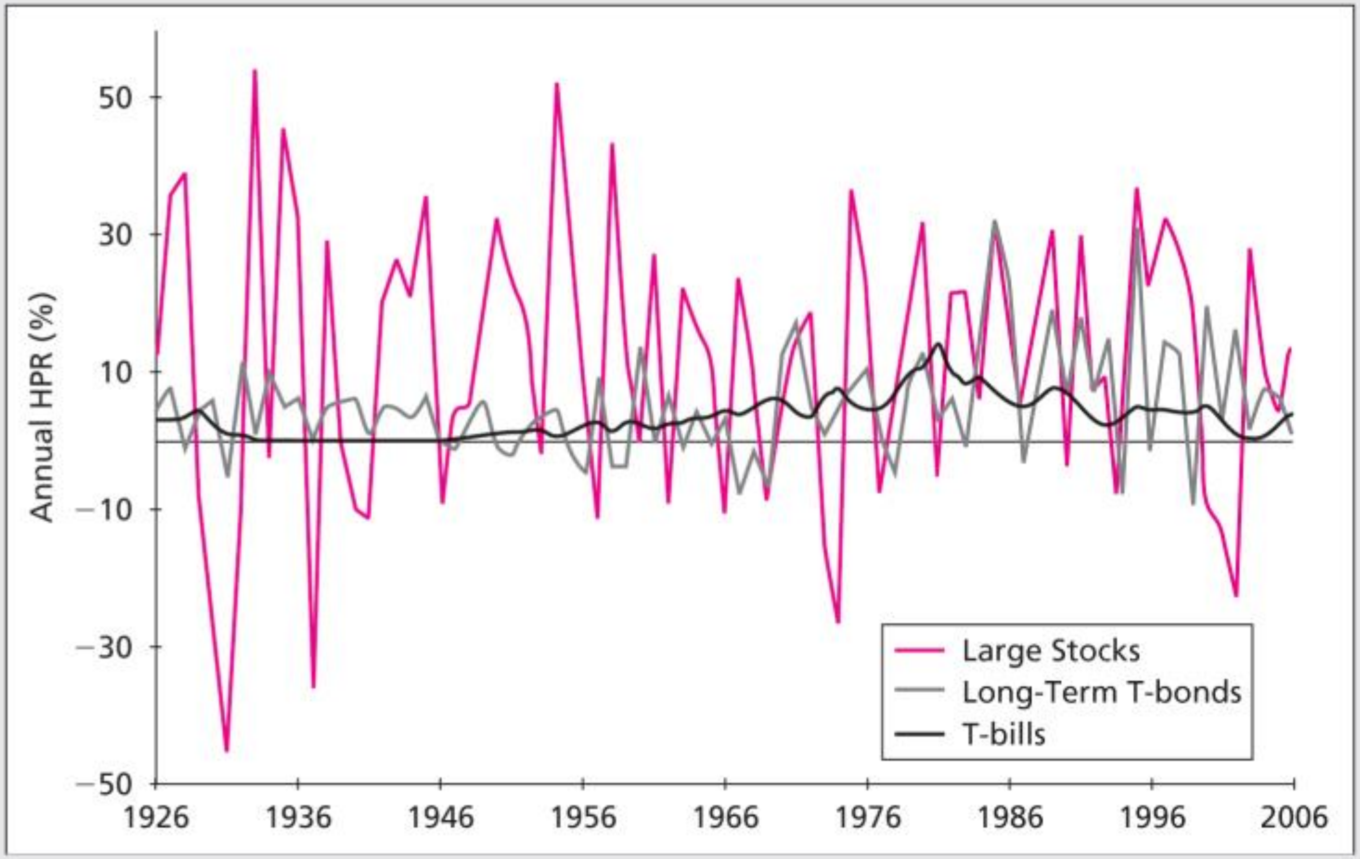


Figure 5.3 Normal Distribution with Mean of 12% and St Dev of 20%

FIGURE 5.3

The normal distribution with mean return 12% and standard deviation 20%

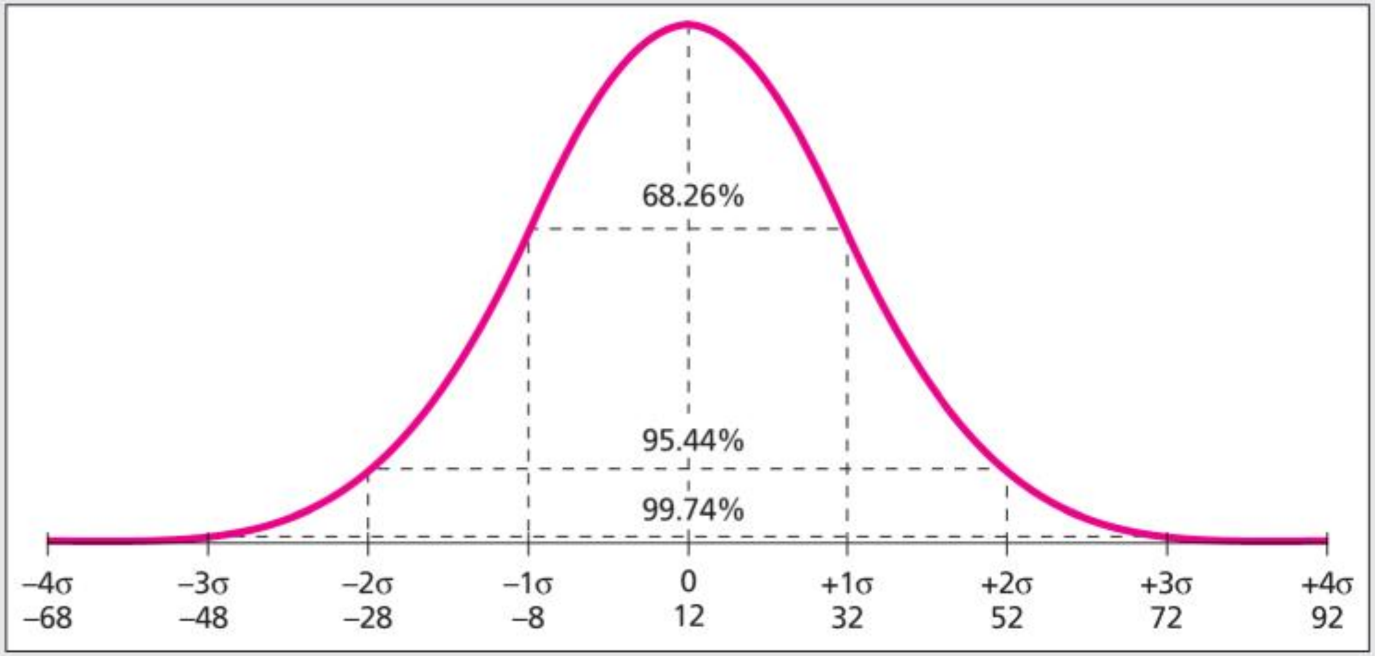


Table 5.4
Size-Decile Portfolios

TABLE 5.4

Size-decile portfolios of the NYSE/AMEX/NASDAQ Summary Statistics of Annual Returns, 1927–2006

Decile	Geometric Average	Arithmetic Average	Standard Deviation
1 Largest	9.6%	11.4%	19.1%
2	10.9	13.2	21.6
3	11.4	13.8	22.9
4	11.9	14.8	25.2
5	12.0	15.2	26.6
6	12.1	15.6	27.6
7	12.4	16.3	30.0
8	12.5	17.0	32.5
9	12.2	17.5	35.3
10 Smallest	13.8	20.4	40.9
Total Value Weighted Index	10.1%	12.1%	20.2%

Source: Web site of Professor Kenneth R. French, **http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html**.

5.4 INFLATION AND REAL RATES OF RETURN

Real vs. Nominal Rates

Fisher effect: Approximation

nominal rate = real rate + inflation premium

$R = r + i$ or $r = R - i$

Example $r = 3\%$, $i = 6\%$

$R = 9\% = 3\% + 6\%$ or $3\% = 9\% - 6\%$

Real vs. Nominal Rates

Fisher effect:

$$1 + r = \frac{1 + R}{1 + i} \text{ or:}$$

$$r = \frac{R - i}{1 + i}$$

2.83% = (9%-6%) / (1.06)

Figure 5.4 Interest, Inflation and Real Rates of Return

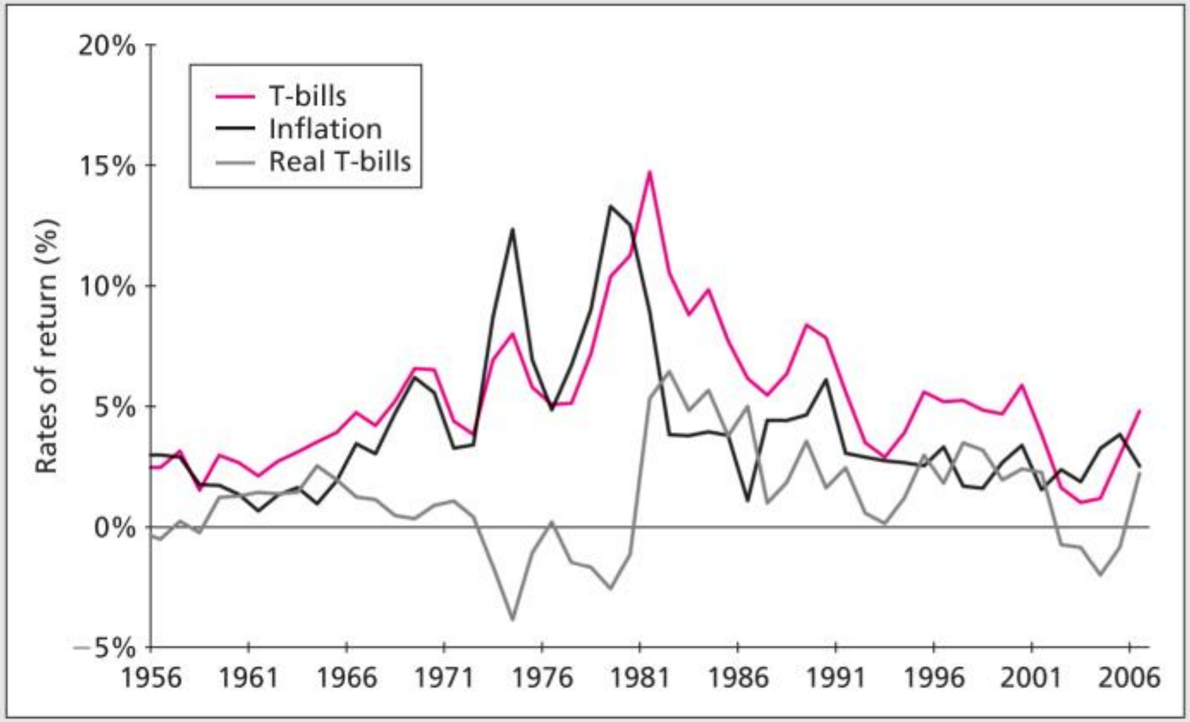


FIGURE 5.4

Interest, inflation, and real rates, 1956–2006

Source: Prepared from data in Table 5.3.

5.5 ASSET ALLOCATION ACROSS RISKY AND RISK-FREE PORTFOLIOS

Allocating Capital

- Possible to split investment funds between safe and risky assets
- Risk free asset: proxy; T-bills
- Risky asset: stock (or a portfolio)

Allocating Capital

- Issues
  - Examine risk/ return tradeoff
  - Demonstrate how different degrees of risk aversion will affect allocations between risky and risk free assets

The Risky Asset: Text Example (Page 143)

Total portfolio value	= $300,000
Risk-free value	= 90,000
Risky (Vanguard and Fidelity)	= 210,000

Vanguard (V) = 54%

Fidelity (F) = 46%

The Risky Asset: Text Example (Page 143)

$$y = \frac{210,000}{300,000} = \quad 0.7(\text{risky assets, portfolio } P)$$

$$1 - y = \frac{90,000}{300,000} = 0.3(\text{risk-free assets})$$

Vanguard	**113,400/300,000 = 0.378**
Fidelity	**96,600/300,000 = 0.322**
Portfolio *P*	**210,000/300,000 = 0.700**
Risk-Free Assets *F*	**90,000/300,000 = 0.300**
Portfolio *C*	**300,000/300,000 = 1.000**

Calculating the Expected Return Text Example (Page 145)

$r_f = 7\%$ $\sigma_{rf} = 0\%$

$E(r_p) = 15\%$ $\sigma_p = 22\%$

$y = \%$ in p $(1-y) = \%$ in r_f

Expected Returns for Combinations

$$E(r_c) = yE(r_p) + (1 - y)r_f$$

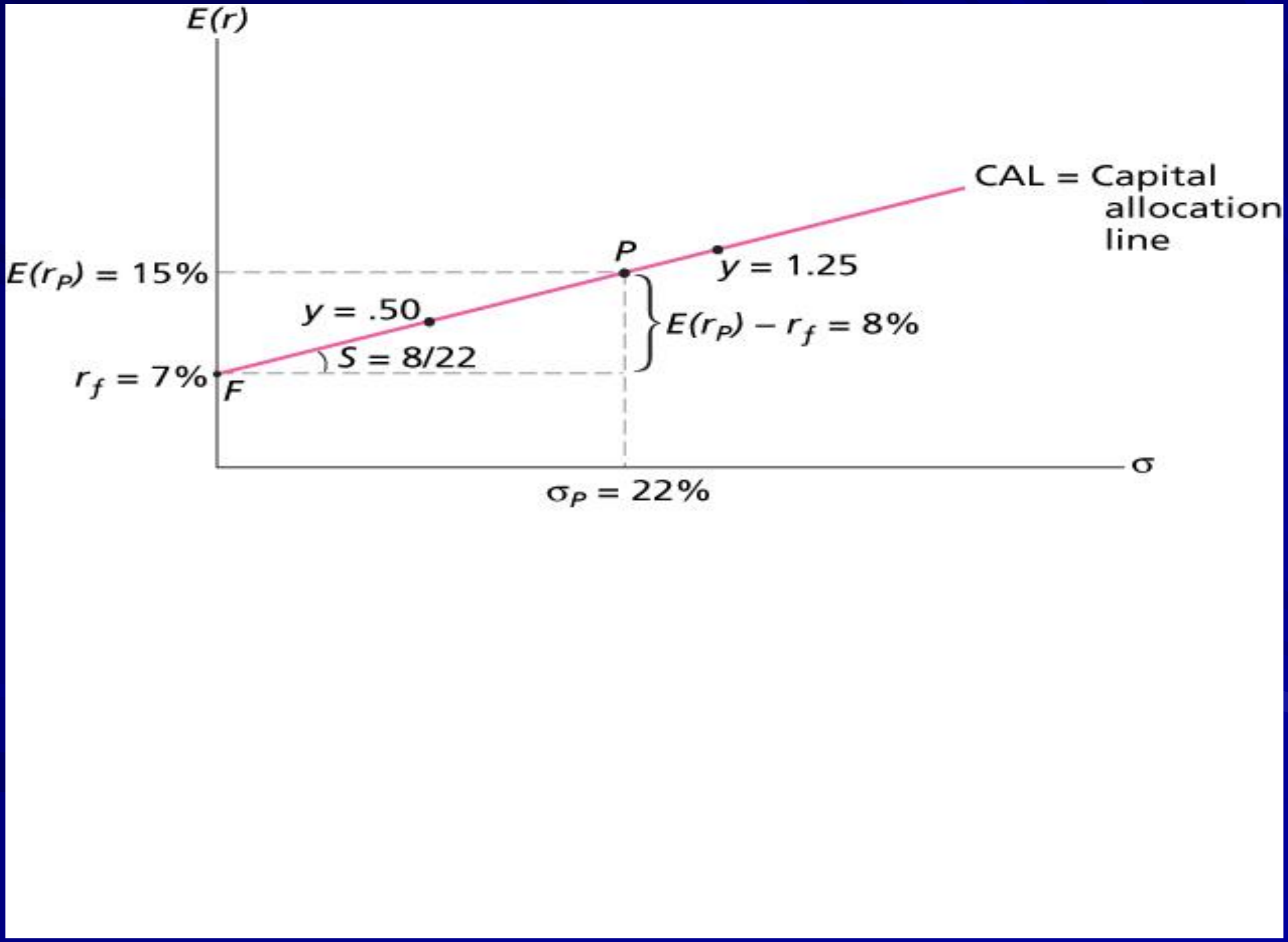
r_c = complete or combined portfolio

For example, $y = .75$

$$E(r_c) = .75(.15) + .25(.07)$$

$$= .13 \text{ or } 13\%$$

Figure 5.5 Investment Opportunity Set with a Risk-Free Investment



Variance on the Possible Combined Portfolios

Since $\sigma_{r_f} = 0$, then

$$\sigma_c = y \sigma_p$$

Combinations Without Leverage

If y = .75, then

σ_c **= .75(.22) = .165 or 16.5%**

If y = 1

σ_c **= 1(.22) = .22 or 22%**

If y = 0

σ_c **= 0(.22) = .00 or 0%**

Using Leverage with Capital Allocation Line

Borrow at the Risk-Free Rate and invest in stock

Using 50% Leverage

$r_c = (-.5)\ (.07) + (1.5)\ (.15) = .19$

$\sigma_c = (1.5)\ (.22) = .33$

Risk Aversion and Allocation

- Greater levels of risk aversion lead to larger proportions of the risk free rate
- Lower levels of risk aversion lead to larger proportions of the portfolio of risky assets
- Willingness to accept high levels of risk for high levels of returns would result in leveraged combinations

5.6 PASSIVE STRATEGIES AND THE CAPITAL MARKET LINE

Table 5.5 Average Rates of Return, Standard Deviation and Reward to Variability

TABLE 5.5

Average excess rate of return, standard deviations and the reward-to-volatility ratio of large common stocks over one-month bills over 1926–2006 and various subperiods

	Excess Return (%)		
	Average	SD	Sharpe Ratio
1926–1946	8.36	27.98	0.30
1947–1966	12.72	18.05	0.70
1967–1986	4.14	17.44	0.24
1987–2006	8.47	16.22	0.52
1926–2006	8.42	20.42	0.41

Source: Data in Table 5.3.

Costs and Benefits of Passive Investing

- Active strategy entails costs
- Free-rider benefit
- Involves investment in two passive portfolios
  - Short-term T-bills
  - Fund of common stocks that mimics a broad market index